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ORIGINAL ARTICLE

Influence of measurement errors on temperature-based death time determination

Michael Hubig · Holger Muggenthaler · Gita Mall

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Abstract Temperature-based methods represent essential tools in forensic death time determination. Empirical double exponential models have gained wide acceptance because they are highly flexible and simple to handle. The most established model commonly used in forensic practice was developed by Henssge. It contains three independent variables: the body mass, the environmental temperature, and the initial body core temperature. The present study investigates the influence of variations in the input data (environmental temperature, initial body core temperature, core temperature, time) on the standard deviation of the model-based estimates of the time since death. Two different approaches were used for calculating the standard deviation: the law of error propagation and the Monte Carlo method. Errors in environmental temperature measurements as well as deviations of the initial rectal temperature were identified as major sources of inaccuracies in model based death time estimation.

Keywords Time since death · Temperature model · Measurement errors

Introduction

Temperature-based death time determination has long been established in medico-legal practice. It is founded on the analysis of postmortem cooling and requires postmortem temperature measurements. Measurements of rectal temperature are most convenient at the crime scene since the measurement location is noninvasively accessible and at the same time provides a core temperature that slowly converges to the ambient temperature [1].

The temperature recordings consist of pairs of measurements, the first measurement denoting the time of temperature recording, t_n, and the second measurement denoting the temperature, T_n. Thus, a finite sequence of N pairs of measurements $(t_n, T_n)_{n=1,\ldots,N}$ is produced. In the extreme case—mostly met in medico-legal practice—N is equal to 1. Temperature-based death time determination also requires approximates of the initial body temperature, T_0, and measurements of the environmental temperature, T_E. Further side conditions, e.g., clothing/covering, convection, and radiation, have to be documented as accurately as possible. If necessary, alternative estimations applying extreme values for the side conditions have to be performed. The model describing postmortem cooling then provides a continuous temperature–time curve $T(t)$, the model curve. It is mostly given by a formula or an algorithm.

Every model-based algorithm for determining the time since death from body temperature decrease can be divided into two steps:

1. Reconstructing the temperature–time curve $T(t, \delta t)$ by deliberate determination of a hypothetical start time δt. Shifting the model curve $T(t, \delta t)$ by all possible time values δt, leading to a family of model curves $(T(t, \delta t))_{\delta t}$.
2. Calculating the most probable time of death from the temperature measurements by selecting an adjusted model curve $T(t_\mathrm{D}*,t)$ from the curve family $(T_\mathrm{R}(t, \delta t))_{\delta t}$,

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M. Hubig · H. Muggenthaler · G. Mall (✉)
Institute of Legal Medicine, University Hospital Jena,
Fürstengraben 23,
07740 Jena, Germany
e-mail: gita.mall@med.uni-jena.de

which means selecting that estimator value t_D* for the time span t_D between death and measurement for which the curve $T(t, t_D^*)$ best fits the data $(t_n, T_n)_{n=1,\ldots,N}$.

In the following, a general scheme for calculating the most probable time since death, t_D*, according to step 2 on the basis of continuous rectal temperature–time curves $(T(t, \delta t))_{\delta t}$ generated by step 1 is described.

General algorithm for back-calculating the time since death

The estimate t_D* of the true time span t_D between death and measurement is determined using the least square method. It is that particular value of the parameter δt that minimizes the sum of the squares of the distances between the measurements $(t_n, T_n)_{n=1,\ldots,N}$ and the curve $T(t, \delta t)$:

$$\min E^2 := \min \sum_{n=1,\ldots,N} (T(t_n, \delta t) - T_n)^2 \rightarrow t_D^*. \tag{1.1}$$

Assuming constant environmental conditions allows the shape of the model curves in the family $(T(t, \delta t))_{\delta t}$ to be determined. The curve graphs are obtained by translation along the time axis from $T(t,0)$:

$$T(t, \delta t) := T(t + \delta t, 0) =: T(t + \delta t). \tag{1.2}$$

The parameter value of δt finally selected by the procedure (Eq. 1.1) is therefore further marked consistently by the symbol t_D*. The parameter δt is called the time shift parameter.

By Eq. 1.2, the algorithm for the death time determination becomes much simpler since in step 1 reconstructing a single temperature–time curve instead of a whole family of curves is sufficient. The algorithm can now be formulated by assembling Eq. 1.1 and Eq. 1.2 as explained in the subsequent sections.

Shift algorithm for back-calculating time of death

The death time estimator t_D* is determined by the least square method. It provides particular value t_D* of the time shift parameter δt, which minimizes the value of the functional E^2:

$$\min E^2 := \min \sum_{n=0,\ldots,N} (T(t_n + \delta t) - T_n)^2 \rightarrow t_D^*. \tag{1.3}$$

Like any other numerical method that calculates output data from input data using a numerical algorithm, the above method (Eq. 1.1–Eq. 1.3) is subject to the following three categories of errors:

A: Errors in the input data
In the case of death time estimation, these are mainly systematic and stochastic errors occurring during the recording of the rectal temperature. Errors in the approximation of the initial body temperature, T_0, or the environmental temperature, T_E, belong to this category of errors as well.

B: Errors in the free parameters of the model
This category comprises errors in determining the free parameters of the model $T(t)$, e.g., errors determining Z and p in the model of Henssge.

C: Systematic errors in the model approach
These errors will occur if the model approach does not sufficiently take into account the relevant aspects of the cooling process.

There are only few studies that investigate the influence of errors on the outcome of temperature-based death time estimation models. Kanawaku et al. [2] studied the effects of rounding errors on postmortem temperature measurements in the external auditory canal caused by thermometer resolution. Other groups compared real cases with exactly known death times to determine the precision of numerical death time estimation models [3–13].

The present paper concentrates on quantifying errors of category A using mathematical formulae. The occurrence of measurement errors in input data cannot be avoided if the measured quantities are real numbers. Their investigation considerably contributes to quantifying the total model error.

Materials and methods

The analysis of the influence of measurement errors of category A is based on the law of error propagation. The calculation process from the input data to the output data is considered as a transformation. The estimation of the standard deviation of the output data with respect to the true values is approximated by Taylor series expansion. So far, the analysis of the influence of measurement errors in model-based death time estimation has not been investigated, probably because an explicit mathematical formulation of the estimator as a function of the input data does not exist. This problem can be solved by two different approaches. First, the well-known implicit functions theorem can be applied, and second, a Monte Carlo simulation can be performed. The present study applies both approaches.

Law of error propagation

The input data vector for death time estimation is denoted by $\boldsymbol{u}$. In the case of N rectal temperature measurements, this vector consists of N rectal temperature–time tuples (t_n, T_n) with $n=1,\ldots,N$, the initial rectal temperature T_0, and the environmental temperature T_E. The algorithm output is the estimated death time t_D*. The time-dependent course of the calculated rectal temperature is denoted by $T(t)$, which cannot always be described by an analytical formula. As a precondition, the temperature curve $T(t)$ is assumed to be a second-order continuously differentiable function defined on the set R of real numbers. The transformation F, which is also called the system function, representing the death time calculation algorithm computes $F(u) := t_D$*. By applying a Taylor series expansion and the implicit function theorem on F, one yields an analytic expression as a linear approximation of small deviations Δt_D* of the estimator t_D* from the true value t_D of death time as a function of small deviations $\Delta \boldsymbol{u}$ of its input vector from its true value $\boldsymbol{u}$. For a detailed mathematical description, see Electronic supplementary materials (ESM).

Stochastic interpretation of the error propagation law

The law of error propagation allows a stochastic interpretation of the variables t_D* and $\boldsymbol{u}$ as random variables. An additive stochastic model is presumed: t_D and $\boldsymbol{u}$ are constant values, but in an additive superposition with the stochastic error variables Δt_D* and Δu. The input vector is therefore given by $\boldsymbol{u} + \Delta \boldsymbol{u}$ and the output value $t_D^* = t_D + \Delta t_D^*$. The probability distributions of the error variables are $P_{\Delta tD*}$ and $P_{\Delta \boldsymbol{u}}$, respectively. Concerning possible error sources, the stochastic parameters $t_D^* := t_D + \Delta t_D^*$ and $\boldsymbol{u} + \Delta \boldsymbol{u}$, the expected values $E(t_D^*)$, $E(\boldsymbol{u} + \Delta \boldsymbol{u})$, the variance $V(t_D^*)$, and the covariance matrix Cov $(\Delta \boldsymbol{u}, \Delta \boldsymbol{u}) = \mathrm{Cov}(\boldsymbol{u} + \Delta \boldsymbol{u}, \boldsymbol{u} + \Delta \boldsymbol{u})$ of the error variables are of major interest. They can now be approximately calculated using the system functions Taylor series expansion. Based on these variances and covariances, the standard deviation $D(t_D^*)$ can be calculated by taking square roots. For a detailed mathematical description, see ESM.

The general contents of the previous sections are valid for every estimation algorithm. Further conclusions can be drawn looking at the structure of a particular algorithm. First formulae for the variance $V(t_D^*)$ and the standard deviation $D(t_D^*)$ are deduced assuming constant environmental conditions. In the general case of changing environmental conditions, analogous formulae can be obtained.

Scaling models

The system function can be further specified only by incorporating characteristics of a particular temperature model $T(t)$. A common class of temperature models are scaling models of the following general form:

$$(T(t) - T_E)/(T_0 - T_E) = f(t). \tag{2.4.1}$$

The function $f(t)$ in Eq. 2.4.1, which is independent of the temperatures T_E and T_0, monotonically decreases with increasing t and takes values between 0 and 1 only. All temperature-based models for death time estimation (e.g., [3, 8–14]) used in forensic routine belong to the class of scaling models.

Double exponential models

Among scaling models, the double exponential models are mainly used in forensic practice [3, 10]. They are highly flexible and relatively simple to handle.

In general, double exponential models can be written as follows:

$$(T(t) - T_E)/(T_0 - T_E) = \alpha \exp(-\beta t) + \gamma \exp(-\delta t). \tag{2.5.1}$$

The rectal temperature is represented by $T(t)$, the environmental temperature by T_E, the body core temperature at death by T_0, and the time of death by t. The exponential terms with their different half-life rates controlled by β and δ are weighted by the parameters α and γ. These exponential terms influence the initial (plateau) and subsequent (exponential) phase of the temperature curve. Double exponential models are parameterized by α, β, γ, and δ.

Due to experimental investigations and physical considerations, Marshall and Hoare [10] were able to narrow the double exponential model to a model containing only two parameters. Based on equation Eq. 2.5.1, Marshall and Hoare introduced two new parameters, p and Z, and defined the parameters α, β, γ, and δ as functions of p and Z.

$$\alpha := p/(p - Z) \tag{2.5.2}$$

$$\gamma := -Z/(p - Z) \tag{2.5.3}$$

$$\beta := Z \tag{2.5.4}$$

$$\delta := p. \tag{2.5.5}$$

With Eqs. 2.5.1–2.5.5 and with the following parameter specification (Eqs. 2.5.6 and 2.5.7), the model of Marshall and Hoare becomes applicable.

$$p = 0.4\ \mathrm{h}^{-1} \tag{2.5.6}$$

$$Z = -0.0573\ \mathrm{h}^{-1} + 0.000625\ S\ \mathrm{kg\ cm}^{-2}\mathrm{h}^{-1}. \tag{2.5.7}$$

S represents the so-called size factor of the human body:

$$S = 0.8\mathrm{A} \ / \ \mathrm{m} \tag{2.5.8}$$

In Eq. 2.5.8, the effective body surface (cm^3) involved in heat loss is represented by A, and m is referred to as the body mass (kg). Marshall and Hoare [15] calibrated their model using 12 cooling experiments and used the remaining numerous experiments for validation.

Further cooling experiments were reported by Henssge [3] who assumed proportionality of body surface and body mass with respect to postmortem cooling behavior. Henssge [3] published a parameter definition that differs from equations Eqs. 2.5.6–2.5.8:

$$Z := 0.0284\ \mathrm{h}^{-1} - 1.2815\ \mathrm{m}^{-0.625}\ \mathrm{h}^{-1}\ \mathrm{kg}^{0.625} \tag{2.5.9}$$

$$p = 5\mathrm{Z} \text{ if } T_{\mathrm{E}} \leq 23.3^{\circ}\mathrm{C} \tag{2.5.10}$$

$$p = 10\mathrm{Z} \text{ if } T_{\mathrm{E}} > 23.3^{\circ}\mathrm{C}. \tag{2.5.11}$$

Using this parameter specification, Henssge introduced his model variant which is well established and commonly used in forensic practice.

By inserting the Henssge model into the equations for the system function, one yields computable expressions for the standard deviation $D(t_{\mathrm{D}}{*})$ of the death time estimator. For a detailed mathematical description, see ESM.

Monte-Carlo simulation

Monte Carlo simulation is another approach for estimating the variance and for verifying the equations presented in the last four sections. A random sample $\left\{u^1, \ldots, u^{N_s}\right\}$ of size N_{S} of the stochastic distribution of input variables is produced using a random generator. The corresponding expected values and variances of the variables u_i have to be selected and serve as moments of the simulated normal input distributions. For every simulated tuple u^i, a death time estimation is performed, resulting in the corresponding death time $t_{\mathrm{D}}{*}^i$. On the basis of the simulated sample $\{t_{\mathrm{D}}{*}^1, \ldots, t_{\mathrm{D}}{*}^{N_{\mathrm{S}}}\}$ of size N_{S}, the standard deviation $D(t_{\mathrm{D}}{*}) = D(\Delta t_{\mathrm{D}}{*}) = D(t_{\mathrm{D}} - t_{\mathrm{D}}{*})$ of the difference $\Delta t_{\mathrm{D}}{*} = t_{\mathrm{D}}{*} - t_{\mathrm{D}}$ can be estimated.

Results

The aim of the analysis performed was to evaluate the error Δt_{D} of the death time estimation process:

$$\Delta t_{\mathrm{D}} := t_{\mathrm{D}}{*} - t_{\mathrm{D}}. \tag{3.1}$$

In all calculations, the standard deviation $D(\Delta t_{\mathrm{D}}{*})$ of the error $\Delta t_{\mathrm{D}}{*}$ which, since the true time of death t_{D} is constant, is identical to the standard deviation $D(t_{\mathrm{D}}{*})$ of the estimator $t_{\mathrm{D}}{*}$ was computed or estimated:

$$D(t_{\mathrm{D}}{*}) = D(\Delta t_{\mathrm{D}}{*}) := V(\Delta t_{\mathrm{D}}{*})^{1/2} = V(t_{\mathrm{D}}{*})^{1/2}. \tag{3.2}$$

The main objective was to investigate the standard deviation $D(\Delta t_{\mathrm{D}}{*})$ as a function of real time since death t_{D}. The curves presented are graphs of the functions

$$D(t_{\mathrm{D}}{*}) = D(\Delta t_{\mathrm{D}}{*}) = D(\Delta t_{\mathrm{D}}{*}, t_{\mathrm{D}}). \tag{3.3}$$

The real time span t_{D} between death and temperature measurement is given by the ordinate in the diagrams. Applying the error propagation law and the Monte Carlo simulation alternatively, each estimation was performed twice:

D_{EP} Standard deviation estimator, law of error propagation
D_{MC} Standard deviation estimator, Monte Carlo method

A sample size of N_{S}=1,000 was defined for the simulation of the Monte Carlo input sample.

Three standard cooling scenarios are cited here as case (a), case (b), and case (c) with the following boundary conditions:

T_{E}=18°C
T_0=37.2°C

In all three cases, the so-called reference standard cooling conditions [3] were assumed, which are:

* Naked body
* Lying on the back, limbs stretched out alongside the body
* On thermally indifferent ground
* No convective cooling by air flow
* Constant environmental temperature

To cope with the influence of body mass, the computations were performed for three different body masses:

(a) m=60 kg
(b) m=80 kg
(c) m=100 kg

These are the cases (a), (b), and (c) mentioned above. The different masses, m, were inserted in the model of Henssge computing D_{EP} as well as D_{MC}. Quantifying the influence of input variable variations on death time estimation output, the parameters T_0, T_{E}, T_{k}, and t_{k} were varied as described below.

The influence of input variable errors was investigated on the one hand for the entire set of input variables and on the other hand for each input variable alone, setting the variances of the other input variables to zero. In detail, this approach results in the following diagrams (note that only the cooling scenario (b) corresponding to the body mass m=80 kg is presented in the diagrams exemplarily) and tables:

- Influence of all input variables: t_k (k=1,…,N); T_k (k=1, …,N); T_0; T_E (Fig. 2 and Table 1)
- Influence of T_0 (Fig. 3 and Table 2)
- Influence of T_E (Fig. 4 and Table 3)
- Influence of T_k (k=1,…,N) (Fig. 5 and Table 4)
- Influence of t_k (k=1,…,N) (Fig. 6 and Table 5)

The values of the estimators D_{EP} and D_{MC} are presented in the diagrams on the abscissae with regard to the following variables:

- The real time span t_D between death and measurement on the ordinates (variation range: t_D=1, 2, 3,…,26 h)
- The number N of body core temperature measurements T_k at times t_k (grading: N=1, N=10, N=100) on a time grid given here without simulated time errors:

$$t_k := t_D + \Delta t\ k \text{ with} : \Delta t := 1\text{min};\ k = 1, \ldots, N$$

Concerning the input variables, the following realistic distributions were presumed:

- Rectal temperature T_0 at time of death:
  normal distribution with expected value $m(T_0)$ and standard deviation $D(T_0)$.

  $$m(T_0) = 37.2°\text{C}; D(T_0) = 0.5°\text{C or } 0°\text{C}.$$

- Environmental temperature T_E during the entire cooling process:
- normal distribution with expected value $m(T_E)$ and standard deviation $D(T_E)$.

  $$m(T_E) = 18.0°\text{C}; D(T_E) = 1°\text{C or } 0°\text{C}.$$

- Rectal temperature T_k at measurement times t_k (k=1,…, N):
  normal distribution with expected value $m(T_k)$ and standard deviation $D(T_k)$.

  $m(T_k)$=Fixed value without variance; $D(T_k)$=0.1°C or 0°C.

- Time value t_k of the measurement times t_k (k=1,…,N): normal distribution with expected value $m(t_k)$ and standard deviation $D(t_k)$.

  $m(t_k) = t_D + k \times 1$ min; $D(t_k)$ = 0.004 h or approx. 15 s.

The practical background of the probability density distributions assumed for the variables T_0, T_E, T_k, and t_k is the following: Every random variable with a normal (Gaussian) distribution with standard deviation D and expected valued E has the property that 68.3%/95.4%/99.7% of all measurement results of this variable lie in the interval (E − D, $E + D$)/($E − 2D$, $E + 2D$)/($E − 3D$, $E + 3D$), respectively.

For the initial rectal temperature, we considered a standard deviation of 0.5°C as appropriate. Consequently, an initial rectal temperature T_0 with an exact value of 37.2°C may lie in the interval of [36.7°C, 37.7°C] in 68.3% of all temperature measurements taken, in the interval of [36.2°C, 38.2°C] in 95.4%, and in the interval of [35.7°C, 38.7°C] in 99.7%.

According to common experience in experimental physics in the temperature range between −20°C and about 100°C, even maximum precision measurements taken in a laboratory may yield standard deviations of about 0.1°C. Since temperature measurements at a crime scene are far from ideal lab conditions, we considered a standard deviation of 1.0°C for the environmental temperature T_E to be appropriate. Additionally, certain conditions at the crime scene influencing the environmental temperature measurement upon arrival of the forensic specialist may have changed (e.g., due to opening of doors and windows). For the environmental temperature T_E, our assumptions mean that measured values of the exact environmental temperature of E = 18.0°C lie with probabilities of 68.3%/95.4%/99.7% within the intervals [17°C, 19°C]/[16°C, 20°C]/[15°C, 21°C], respectively.

The measurements of the rectal temperature T_k at a fixed location are subject only to measurement errors due to instrumental noise or inaccurate registration. In the case of the rectal temperature measurements T_k at the times t_k, our assumption of a standard deviation of 0.1°C leads to probabilities of 68.3%/95.4%/99.7% for temperature intervals of $8T_{k,\text{exact}}$ − 0.1°C, $T_{k,\text{exact}}$ + 0.1°C]/[$T_{k,\text{exact}}$ − 0.2°C, $T_{k,\text{exact}}$ + 0.2°C]/[$T_{k,\text{exact}}$ − 0.3°C, $T_{k,\text{exact}}$ + 0.3°C], respectively.

For the time value t_k, a standard deviation of D=0.004 h=14.4 s was assumed. Since time measurement—in contrast to medium range temperature measurement—can be performed with stunning accuracy, this D value appears to be appropriate. Taking into account real crime scene work, which includes taking the value t_k by reading a watch as well as registering the time a posteriori, the actual error may be underestimated. At the crime scene, the usual proceeding is to take only one time t_1 and one rectal temperature T_1. Common sense as well as the formulae tell us that in this case, any error in t_1 registration leads to exactly the same error in death time t_D calculation. This makes it easy for the investigator to control and take into account his own time measurement errors.

Further essential error sources in temperature-based death time determination are errors in taking the body mass. By inspection of Henssge's formulae (Eqs. 2.5.9–2.5.11) and the double exponential model (Eqs. 2.5.1–

Table 1 Case (a): standard deviation D_{EP} (computed according to the law of error propagation) and D_{MC} (computed by Monte Carlo simulation) as functions of time t_D between death and measurement under the overall influence of the confounders T_0, T_E, T_k, and t_k

t_D [h]	m=60kg						m=80kg						m=100kg					
	D_{EP} 1 [h]	D_{MC} 1 [h]	D_{EP} 10 [h]	D_{MC} 10 [h]	D_{EP} 100 [h]	D_{MC} 100 [h]	D_{EP} 1 [h]	D_{MC} 1 [h]	D_{EP} 10 [h]	D_{MC} 10 [h]	D_{EP} 100 [h]	D_{MC} 100 [h]	D_{EP} 1 [h]	D_{MC} 1 [h]	D_{EP} 10 [h]	D_{MC} 10 [h]	D_{EP} 100 [h]	D_{MC} 100 [h]
1	1.29	0.93	1.20	0.94	0.79	0.76	2.09	1.14	1.93	1.16	1.24	1.03	3.16	1.38	2.91	1.39	1.81	1.30
2	0.77	0.92	0.74	0.86	0.62	0.65	1.21	1.22	1.15	1.18	0.93	0.99	1.77	1.49	1.69	1.47	1.34	1.33
3	0.61	0.66	0.59	0.63	0.54	0.55	0.92	1.06	0.89	1.02	0.78	0.82	1.32	1.46	1.27	1.43	1.10	1.18
4	0.54	0.51	0.52	0.52	0.50	0.50	0.78	0.76	0.76	0.78	0.71	0.70	1.10	1.15	1.07	1.15	0.97	0.99
5	0.51	0.50	0.50	0.51	0.49	0.50	0.71	0.70	0.69	0.71	0.66	0.67	0.97	0.99	0.95	1.00	0.89	0.92
6	0.51	0.52	0.49	0.50	0.50	0.51	0.67	0.69	0.66	0.66	0.64	0.65	0.90	0.94	0.88	0.89	0.84	0.86
7	0.52	0.53	0.51	0.52	0.53	0.54	0.66	0.67	0.64	0.64	0.64	0.66	0.85	0.88	0.83	0.83	0.81	0.84
8	0.55	0.59	0.54	0.54	0.57	0.58	0.66	0.71	0.64	0.65	0.65	0.66	0.83	0.89	0.81	0.82	0.80	0.80
9	0.59	0.61	0.58	0.59	0.62	0.63	0.67	0.69	0.66	0.67	0.68	0.68	0.82	0.84	0.80	0.83	0.80	0.80
10	0.64	0.65	0.64	0.64	0.68	0.67	0.70	0.71	0.69	0.70	0.71	0.70	0.82	0.84	0.80	0.83	0.81	0.80
11	0.71	0.74	0.70	0.68	0.75	0.76	0.73	0.76	0.72	0.71	0.76	0.76	0.83	0.85	0.82	0.81	0.83	0.84
12	0.78	0.79	0.77	0.80	0.83	0.85	0.78	0.78	0.77	0.79	0.81	0.82	0.86	0.85	0.84	0.86	0.86	0.87
13	0.87	0.87	0.86	0.89	0.93	0.99	0.83	0.83	0.82	0.84	0.87	0.92	0.89	0.88	0.87	0.88	0.90	0.96
14	0.96	0.98	0.95	0.98	1.03	1.07	0.90	0.91	0.89	0.89	0.94	0.97	0.93	0.93	0.91	0.91	0.95	0.97
15	1.06	1.09	1.06	1.06	1.14	1.20	0.97	0.98	0.96	0.95	1.01	1.04	0.97	0.98	0.96	0.94	1.00	1.01
16	1.18	1.13	1.18	1.25	1.26	1.28	1.05	1.01	1.04	1.10	1.10	1.10	1.03	1.00	1.01	1.06	1.06	1.05
17	1.31	1.32	1.30	1.31	1.40	1.49	1.13	1.14	1.12	1.12	1.19	1.23	1.09	1.11	1.08	1.07	1.12	1.14
18	1.44	1.45	1.44	1.47	1.54	1.61	1.23	1.22	1.22	1.22	1.29	1.30	1.16	1.15	1.14	1.14	1.19	1.18
19	1.59	1.60	1.59	1.65	1.70	1.77	1.33	1.32	1.32	1.34	1.40	1.42	1.23	1.21	1.22	1.22	1.27	1.28
20	1.75	1.85	1.75	1.93	1.87	1.98	1.44	1.50	1.43	1.53	1.52	1.56	1.31	1.34	1.30	1.37	1.36	1.38
21	1.93	1.92	1.92	2.07	2.06	2.28	1.56	1.51	1.55	1.62	1.64	1.76	1.40	1.34	1.38	1.43	1.45	1.54
22	2.11	2.43	2.11	2.16	2.25	2.33	1.68	1.85	1.68	1.66	1.77	1.76	1.49	1.61	1.48	1.44	1.55	1.52
23	2.31	2.56	2.31	2.44	2.47	2.71	1.82	1.93	1.81	1.84	1.91	2.00	1.59	1.66	1.57	1.58	1.65	1.71
24	2.53	2.67	2.53	2.89	2.70	3.14	1.96	1.98	1.96	2.10	2.06	2.25	1.69	1.69	1.68	1.77	1.76	1.88
25	2.77	3.06	2.77	3.08	2.94	3.27	2.12	2.21	2.11	2.23	2.22	2.28	1.80	1.85	1.79	1.87	1.87	1.87

The numbers, N, of measurements N=1, N=10, and N=100 are shown directly below the symbols D_{EP} and D_{MC}

Table 2 Standard deviation D_{EP} (computed according to the law of error propagation) and D_{MC} (computed by Monte Carlo simulation) as functions of time t_D between death and measurement under the influence of the confounder rectal temperature at death time T_0

t_D [h]	m=60kg						m=80kg						m=100kg					
	D_{EP} 1 [h]	D_{MC} 1 [h]	D_{EP} 10 [h]	D_{MC} 10 [h]	D_{EP} 100 [h]	D_{MC} 100 [h]	D_{EP} 1 [h]	D_{MC} 1 [h]	D_{EP} 10 [h]	D_{MC} 10 [h]	D_{EP} 100 [h]	D_{MC} 100 [h]	D_{EP} 1 [h]	D_{MC} 1 [h]	D_{EP} 10 [h]	D_{MC} 10 [h]	D_{EP} 100 [h]	D_{MC} 100 [h]
1	1.27	0.91	1.19	0.89	0.79	0.75	2.05	1.11	1.93	1.11	1.23	1.02	3.10	1.34	2.90	1.33	1.81	1.29
2	0.75	0.87	0.74	0.85	0.61	0.65	1.18	1.20	1.15	1.20	0.93	0.98	1.74	1.46	1.69	1.48	1.34	1.33
3	0.59	0.60	0.58	0.62	0.52	0.54	0.89	0.99	0.88	0.98	0.77	0.81	1.29	1.41	1.27	1.37	1.10	1.18
4	0.51	0.53	0.50	0.52	0.47	0.48	0.75	0.81	0.75	0.80	0.69	0.70	1.07	1.22	1.06	1.19	0.96	1.02
5	0.46	0.46	0.46	0.46	0.44	0.44	0.67	0.67	0.67	0.67	0.63	0.62	0.94	0.96	0.93	0.96	0.87	0.88
6	0.43	0.43	0.43	0.43	0.42	0.44	0.62	0.62	0.62	0.61	0.59	0.61	0.85	0.86	0.85	0.85	0.81	0.85
7	0.42	0.40	0.41	0.41	0.40	0.41	0.58	0.56	0.58	0.59	0.56	0.56	0.80	0.76	0.79	0.80	0.76	0.77
8	0.40	0.42	0.40	0.40	0.39	0.39	0.56	0.58	0.56	0.56	0.54	0.54	0.75	0.78	0.75	0.76	0.73	0.73
9	0.39	0.40	0.39	0.40	0.39	0.39	0.54	0.55	0.54	0.55	0.53	0.52	0.72	0.73	0.72	0.74	0.70	0.70
10	0.39	0.40	0.39	0.37	0.38	0.38	0.53	0.54	0.53	0.51	0.52	0.51	0.70	0.72	0.70	0.68	0.68	0.68
11	0.38	0.37	0.38	0.37	0.38	0.38	0.52	0.50	0.52	0.50	0.51	0.51	0.68	0.66	0.68	0.66	0.67	0.68
12	0.38	0.38	0.38	0.39	0.38	0.38	0.51	0.51	0.51	0.52	0.50	0.50	0.66	0.67	0.66	0.68	0.65	0.66
13	0.38	0.36	0.38	0.38	0.37	0.37	0.50	0.48	0.50	0.51	0.50	0.49	0.65	0.62	0.65	0.66	0.64	0.64
14	0.37	0.38	0.37	0.37	0.37	0.36	0.50	0.50	0.50	0.49	0.49	0.48	0.64	0.65	0.64	0.63	0.64	0.62
15	0.37	0.39	0.37	0.37	0.37	0.36	0.49	0.51	0.49	0.49	0.49	0.48	0.63	0.66	0.63	0.63	0.63	0.62
16	0.37	0.36	0.37	0.36	0.37	0.37	0.49	0.48	0.49	0.48	0.49	0.48	0.63	0.61	0.63	0.61	0.62	0.62
17	0.37	0.37	0.37	0.37	0.37	0.38	0.49	0.48	0.49	0.49	0.49	0.49	0.62	0.62	0.62	0.63	0.62	0.63
18	0.37	0.37	0.37	0.38	0.37	0.38	0.49	0.48	0.49	0.50	0.48	0.50	0.62	0.61	0.62	0.63	0.62	0.64
19	0.37	0.37	0.37	0.38	0.37	0.37	0.48	0.49	0.48	0.47	0.48	0.48	0.61	0.62	0.61	0.59	0.61	0.61
20	0.37	0.36	0.37	0.36	0.37	0.38	0.48	0.47	0.48	0.47	0.48	0.49	0.61	0.59	0.61	0.59	0.61	0.62
21	0.37	0.38	0.37	0.36	0.37	0.37	0.48	0.50	0.48	0.47	0.48	0.48	0.61	0.63	0.61	0.59	0.61	0.62
22	0.37	0.37	0.37	0.37	0.37	0.38	0.48	0.48	0.48	0.48	0.48	0.49	0.61	0.61	0.61	0.61	0.61	0.62
23	0.37	0.37	0.37	0.36	0.37	0.36	0.48	0.48	0.48	0.48	0.48	0.47	0.61	0.61	0.61	0.60	0.60	0.60
24	0.37	0.38	0.37	0.37	0.37	0.36	0.48	0.50	0.48	0.48	0.48	0.47	0.60	0.63	0.60	0.60	0.60	0.60
25	0.37	0.38	0.37	0.37	0.37	0.38	0.47	0.50	0.48	0.48	0.48	0.50	0.60	0.62	0.60	0.60	0.60	0.63

The numbers, N, of measurements N=1, N=10, and N=100 are shown directly below the symbols $D_{EP,N}$ and $D_{MC,N}$

Table 3 Standard deviation D_{EP} (computed according to the law of error propagation) and D_{MC} (computed by Monte Carlo simulation) as functions of time t_D between death and measurement under the influence of the confounder environmental temperature at death time T_E

t_D [h]	m=60kg						m=80kg						m=100kg					
	D_{EP} 1 [h]	D_{MC} 1 [h]	D_{EP} 10 [h]	D_{MC} 10 [h]	D_{EP} 100 [h]	D_{MC} 100 [h]	D_{EP} 1 [h]	D_{MC} 1 [h]	D_{EP} 10 [h]	D_{MC} 10 [h]	D_{EP} 100 [h]	D_{MC} 100 [h]	D_{EP} 1 [h]	D_{MC} 1 [h]	D_{EP} 10 [h]	D_{MC} 10 [h]	D_{EP} 100 [h]	D_{MC} 100 [h]
1	0.03	0.03	0.03	0.03	0.06	0.06	0.03	0.03	0.03	0.03	0.06	0.05	0.03	0.03	0.03	0.03	0.06	0.06
2	0.06	0.06	0.06	0.06	0.09	0.09	0.06	0.06	0.06	0.06	0.09	0.08	0.06	0.06	0.06	0.06	0.09	0.09
3	0.10	0.10	0.10	0.10	0.13	0.14	0.09	0.09	0.09	0.09	0.12	0.13	0.09	0.09	0.09	0.09	0.12	0.13
4	0.14	0.13	0.14	0.14	0.18	0.17	0.13	0.13	0.13	0.13	0.16	0.16	0.12	0.12	0.13	0.12	0.16	0.16
5	0.18	0.19	0.19	0.18	0.23	0.23	0.17	0.18	0.17	0.17	0.21	0.20	0.16	0.17	0.16	0.16	0.20	0.20
6	0.23	0.24	0.24	0.24	0.28	0.29	0.21	0.22	0.22	0.22	0.25	0.25	0.20	0.21	0.20	0.21	0.24	0.24
7	0.29	0.29	0.30	0.29	0.34	0.35	0.26	0.26	0.27	0.26	0.31	0.30	0.24	0.24	0.25	0.24	0.28	0.29
8	0.35	0.37	0.36	0.38	0.41	0.42	0.32	0.33	0.32	0.34	0.36	0.37	0.29	0.31	0.30	0.31	0.33	0.34
9	0.42	0.43	0.43	0.43	0.48	0.49	0.37	0.38	0.38	0.38	0.42	0.42	0.34	0.35	0.35	0.34	0.38	0.39
10	0.50	0.48	0.50	0.51	0.56	0.57	0.43	0.42	0.44	0.45	0.49	0.49	0.39	0.38	0.40	0.40	0.44	0.44
11	0.58	0.62	0.59	0.60	0.65	0.67	0.50	0.53	0.50	0.52	0.56	0.56	0.45	0.48	0.46	0.47	0.50	0.51
12	0.67	0.67	0.67	0.68	0.74	0.74	0.57	0.57	0.58	0.58	0.63	0.62	0.51	0.51	0.52	0.51	0.56	0.56
13	0.76	0.77	0.77	0.78	0.85	0.88	0.65	0.65	0.65	0.65	0.71	0.72	0.57	0.58	0.58	0.58	0.63	0.64
14	0.87	0.89	0.88	0.90	0.96	0.99	0.73	0.74	0.73	0.75	0.80	0.81	0.64	0.65	0.65	0.66	0.70	0.71
15	0.98	1.02	0.99	0.97	1.08	1.07	0.81	0.83	0.82	0.80	0.89	0.87	0.71	0.73	0.72	0.70	0.77	0.76
16	1.10	1.07	1.11	1.20	1.21	1.24	0.91	0.87	0.91	0.97	0.98	0.98	0.79	0.76	0.79	0.84	0.85	0.85
17	1.24	1.23	1.25	1.30	1.35	1.48	1.00	0.99	1.01	1.04	1.09	1.16	0.87	0.85	0.87	0.89	0.94	0.99
18	1.38	1.38	1.39	1.44	1.50	1.57	1.11	1.09	1.12	1.13	1.20	1.22	0.95	0.93	0.96	0.96	1.02	1.04
19	1.53	1.53	1.54	1.63	1.66	1.74	1.22	1.20	1.23	1.27	1.31	1.34	1.04	1.01	1.05	1.08	1.12	1.13
20	1.70	1.80	1.71	1.84	1.84	1.84	1.34	1.39	1.35	1.41	1.44	1.41	1.13	1.17	1.14	1.18	1.21	1.19
21	1.87	1.88	1.89	1.96	2.02	2.14	1.46	1.44	1.47	1.48	1.57	1.60	1.23	1.20	1.24	1.24	1.31	1.33
22	2.06	2.35	2.08	2.24	2.22	2.54	1.59	1.73	1.60	1.67	1.71	1.84	1.33	1.42	1.34	1.38	1.42	1.51
23	2.27	2.50	2.28	2.63	2.44	2.66	1.73	1.83	1.74	1.89	1.85	1.91	1.44	1.50	1.45	1.55	1.53	1.56
24	2.48	2.63	2.50	2.87	2.67	3.04	1.88	1.92	1.89	2.03	2.01	2.14	1.56	1.57	1.56	1.65	1.65	1.73
25	2.72	2.99	2.74	3.02	2.92	3.78	2.04	2.13	2.05	2.12	2.17	2.39	1.67	1.72	1.68	1.71	1.77	1.90

The numbers, N, of measurements $N=1$, $N=10$, and $N=100$ are shown directly below the symbols D_{EP} and D_{MC}

Table 4 Standard deviation D_{EP} (computed according to the law of error propagation) and D_{MC} (computed by Monte Carlo simulation) as functions of time t_D between death and measurement under the influence of the confounder measured rectal temperature T_k

t_D [h]	m=60kg						m=80kg						m=100kg					
	D_{EP} 1 [h]	D_{MC} 1 [h]	D_{EP} 10 [h]	D_{MC} 10 [h]	D_{EP} 100 [h]	D_{MC} 100 [h]	D_{EP} 1 [h]	D_{MC} 1 [h]	D_{EP} 10 [h]	D_{MC} 10 [h]	D_{EP} 100 [h]	D_{MC} 100 [h]	D_{EP} 1 [h]	D_{MC} 1 [h]	D_{EP} 10 [h]	D_{MC} 10 [h]	D_{EP} 100 [h]	D_{MC} 100 [h]
1	0.26	0.52	0.08	0.08	0.02	0.02	0.41	0.65	0.12	0.21	0.03	0.02	0.62	0.72	0.18	0.39	0.04	0.04
2	0.16	0.16	0.05	0.05	0.01	0.01	0.24	0.25	0.07	0.08	0.02	0.02	0.35	0.39	0.11	0.11	0.03	0.03
3	0.13	0.13	0.04	0.04	0.01	0.01	0.19	0.19	0.06	0.06	0.02	0.02	0.27	0.27	0.08	0.08	0.02	0.02
4	0.12	0.11	0.04	0.04	0.01	0.01	0.16	0.16	0.05	0.05	0.02	0.02	0.23	0.22	0.07	0.07	0.02	0.02
5	0.11	0.11	0.03	0.04	0.01	0.01	0.15	0.15	0.05	0.05	0.01	0.01	0.20	0.21	0.06	0.07	0.02	0.02
6	0.11	0.11	0.03	0.04	0.01	0.01	0.15	0.15	0.05	0.05	0.01	0.01	0.19	0.19	0.06	0.06	0.02	0.02
7	0.11	0.11	0.04	0.04	0.01	0.01	0.14	0.15	0.05	0.05	0.01	0.01	0.18	0.19	0.06	0.06	0.02	0.02
8	0.12	0.12	0.04	0.04	0.01	0.01	0.14	0.15	0.05	0.04	0.01	0.01	0.18	0.18	0.06	0.06	0.02	0.02
9	0.12	0.12	0.04	0.04	0.01	0.01	0.15	0.15	0.05	0.05	0.01	0.01	0.18	0.18	0.06	0.06	0.02	0.02
10	0.13	0.13	0.04	0.04	0.01	0.01	0.15	0.15	0.05	0.05	0.02	0.02	0.18	0.18	0.06	0.06	0.02	0.02
11	0.13	0.13	0.04	0.04	0.01	0.01	0.15	0.15	0.05	0.05	0.02	0.02	0.18	0.17	0.06	0.06	0.02	0.02
12	0.14	0.14	0.05	0.05	0.01	0.01	0.16	0.15	0.05	0.05	0.02	0.02	0.18	0.18	0.06	0.06	0.02	0.02
13	0.15	0.15	0.05	0.05	0.02	0.02	0.17	0.16	0.05	0.05	0.02	0.02	0.19	0.19	0.06	0.06	0.02	0.02
14	0.16	0.16	0.05	0.05	0.02	0.02	0.17	0.17	0.05	0.05	0.02	0.02	0.19	0.19	0.06	0.06	0.02	0.02
15	0.17	0.18	0.05	0.06	0.02	0.02	0.18	0.19	0.06	0.06	0.02	0.02	0.20	0.21	0.06	0.06	0.02	0.02
16	0.18	0.18	0.06	0.06	0.02	0.02	0.19	0.19	0.06	0.06	0.02	0.02	0.20	0.20	0.06	0.06	0.02	0.02
17	0.20	0.20	0.06	0.06	0.02	0.02	0.20	0.21	0.06	0.06	0.02	0.02	0.21	0.22	0.07	0.07	0.02	0.02
18	0.21	0.21	0.07	0.07	0.02	0.02	0.21	0.21	0.07	0.07	0.02	0.02	0.22	0.22	0.07	0.07	0.02	0.02
19	0.23	0.23	0.07	0.07	0.02	0.02	0.22	0.22	0.07	0.07	0.02	0.02	0.23	0.23	0.07	0.07	0.02	0.02
20	0.24	0.24	0.08	0.08	0.03	0.03	0.23	0.23	0.07	0.07	0.02	0.02	0.24	0.24	0.07	0.08	0.02	0.02
21	0.26	0.26	0.08	0.08	0.03	0.03	0.24	0.24	0.08	0.08	0.03	0.03	0.24	0.24	0.08	0.08	0.03	0.03
22	0.28	0.28	0.09	0.09	0.03	0.03	0.26	0.26	0.08	0.08	0.03	0.03	0.25	0.26	0.08	0.08	0.03	0.03
23	0.30	0.30	0.10	0.09	0.03	0.03	0.27	0.27	0.09	0.08	0.03	0.03	0.27	0.26	0.08	0.08	0.03	0.03
24	0.32	0.33	0.10	0.10	0.03	0.03	0.28	0.29	0.09	0.09	0.03	0.03	0.28	0.28	0.09	0.09	0.03	0.03
25	0.35	0.36	0.10	0.11	0.04	0.04	0.30	0.31	0.10	0.10	0.03	0.03	0.29	0.30	0.09	0.09	0.03	0.03

The numbers, N, of measurements N=1, N=10, and N=100 are shown directly below the symbols D_{EP} and D_{MC}

Table 5 Standard deviation D_{EP} (computed according to the law of error propagation) and D_{MC} (computed by Monte Carlo simulation) as functions of time t_D between death and measurement under the influence of the confounder measured time t_k

t_D [h]	m=60kg		m=80kg		m=100kg	
	D_{EP} 1 [h]	D_{MC} 1 [h]	D_{EP} 10 [h]	D_{MC} 10 [h]	D_{EP} 100 [h]	D_{MC} 100 [h]
1.00000	0.0040	0.0040	0.0013	0.0013	0.0004	0.0004
2.00000	0.0040	0.0041	0.0013	0.0012	0.0004	0.0004
3.00000	0.0040	0.0039	0.0013	0.0012	0.0004	0.0004
4.00000	0.0040	0.0041	0.0013	0.0013	0.0004	0.0004
5.00000	0.0040	0.0040	0.0013	0.0012	0.0004	0.0004
6.00000	0.0040	0.0040	0.0013	0.0013	0.0004	0.0004
7.00000	0.0040	0.0041	0.0013	0.0013	0.0004	0.0004
8.00000	0.0040	0.0042	0.0013	0.0013	0.0004	0.0004
9.00000	0.0040	0.0040	0.0013	0.0013	0.0004	0.0004
10.00000	0.0040	0.0040	0.0013	0.0012	0.0004	0.0004
11.00000	0.0040	0.0038	0.0013	0.0012	0.0004	0.0004
12.00000	0.0040	0.0040	0.0013	0.0013	0.0004	0.0004
13.00000	0.0040	0.0042	0.0013	0.0013	0.0004	0.0004
14.00000	0.0040	0.0041	0.0013	0.0013	0.0004	0.0004
15.00000	0.0040	0.0040	0.0013	0.0013	0.0004	0.0004
16.00000	0.0040	0.0041	0.0013	0.0013	0.0004	0.0004
17.00000	0.0040	0.0040	0.0013	0.0013	0.0004	0.0004
18.00000	0.0040	0.0040	0.0013	0.0012	0.0004	0.0004
19.00000	0.0040	0.0039	0.0013	0.0013	0.0004	0.0004
20.00000	0.0040	0.0040	0.0013	0.0013	0.0004	0.0004
21.00000	0.0040	0.0040	0.0013	0.0013	0.0004	0.0004
22.00000	0.0040	0.0041	0.0013	0.0013	0.0004	0.0004
23.00000	0.0040	0.0042	0.0013	0.0013	0.0004	0.0004
24.00000	0.0040	0.0042	0.0013	0.0013	0.0004	0.0004
25.00000	0.0040	0.0040	0.0013	0.0013	0.0004	0.0004

The numbers, N, of measurements N=1, N=10, and N=100 are shown directly below the symbols D_{EP} and D_{MC}. The table contains no separate compartments for the three cases of body weight: (a) m=60 kg, (b) m=80 kg, (c) m = 100 kg since maximal differences in these cases are of order 10^{-5} h

2.5.5), it can easily be noticed that the body mass m appears in the formulae in a distinctly nonlinear way. This fact seems to exclude an analogous approach like for the other variables T_0, T_E, T_k, and t_k by comparing a Taylor series expansion of order 1 to a Monte Carlo simulation. It might be necessary to use a Taylor series expansion up to the second, and maybe to the third, order. Since the mathematical framework of this approach is much more complicated than the one used for the variables T_0, T_E, T_k, and t_k and since the body mass m is often estimated at the crime scene as a first guess and later on exactly measured by weighing the corpse, the influence of an erroneous body mass determination was not included in the present study. To cope with the influence of the value of m (not of the error of m) on the input–output relations of the errors of the variables T_0, T_E, T_k, and t_k on the error of t_D, the three scenarios (a), (b), and (c) with different body masses were computed since due to the nonlinearity of m in the error expressions, no scaling relation exists. By carful inter- and extrapolation of the standard deviation D of the three cases (a), (b), and (c), valid numbers for other body masses can be obtained.

The Marshal and Hoare model (Eqs. 2.5.1–2.5.5) with Henssge's formulae (Eqs. 2.5.9, 2.5.10, and 2.5.11) for the determination of the parameters p and Z was further adjusted to cope with non-standard environmental conditions by introducing the so-called corrective factor c of the body mass m starting from the gross assumption that for any non-standard cooling situation with cooling curve $T(t)$ and a body of mass m, a different body mass m' can be calculated such that the standard cooling curve $T'(t)$ of a body of mass m' is identical to $T(t)$. The quotient $c = m/m'$ is the corrective factor and listed in tables for different non-standard conditions up to the first decimal. If this factor c is included in Henssge's formula (Eq. 2.5.9) for Z, the term "m" has to be substituted by "$m \times c$". As one can see by a simple calculation, numerator and denominator of the expressions 2.5.2 and 2.5.3 are multiples of the variable Z. Therefore, the factors α in Eq. 2.5.2 and γ in Eq. 2.5.4 do not depend on Z or m or c at all. It is only via the argument of the exponential function in Eq. 2.5.1 that Z, m, or c do influence the rectal temperature value $T(t)$ at time t. Thus, the above statements for the body mass m are valid for the correction factor c as well. The corrective factor c neither represents a continuous variable nor can it be measured. Consequently, it remains unclear which probability density function has to be assigned to the random variable c. Both our approaches, Monte Carlo simulation as well as the law of error propagation, rely on the known probability density functions of the input variables. Thus, the corrective factor c is not included in the present investigation.

Figure 1 exemplarily shows the Henssge model simulated rectal temperatures for a body mass m=60 kg/80 kg/100 kg, an initial temperature T_0=37.2°C, and an environmental temperature T_E=18°C.

The combined influence of errors in all input parameters

The time variable t_D of the ordinate of the diagrams in Figs. 1, 2, 3, 4, 5, and 6 is defined as the time span between death and the moment when the first temperature measurement is recorded.

The combined influence of all confounders on the standard deviation D is illustrated in Fig. 2 for case (b); the data for all three cases are listed in Table 1. Results D_{EP} obtained by the law of error propagation are represented by solid lines, and the standard deviations D_{MC} calculated via Monte Carlo simulation are represented by dashed lines. Each solid line shows the standard deviation for a distinct number of simulated rectal temperature measurements: at t=1 h: upper curve N=1 ($D_{EP,1}$), mid-curve N=10 ($D_{EP,10}$), lower curve N=100 ($D_{EP,100}$). Concerning the results of the Monte Carlo simulation, the curves can be interpreted as follows: long dashes N=1 ($D_{MC,1}$), short dashes N=10 ($D_{MC,10}$), dots N=100 ($D_{MC,100}$).

As a consistent characteristic of all curves, a significant decrease in the standard deviations during t_D=1−2 h (phase 1) can be observed with values of D between t_D=0.7 h and

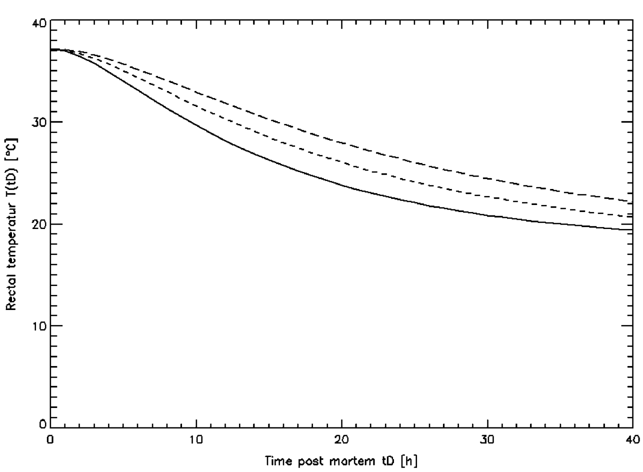

Fig. 1 Simulated model curve $T(t_D)$ according to Henssge's model with T_0=37.2°C, T_E=18°C for a body mass m=60 kg (*line*), 80 kg (*dashed*), 100 kg (*long dashes*)

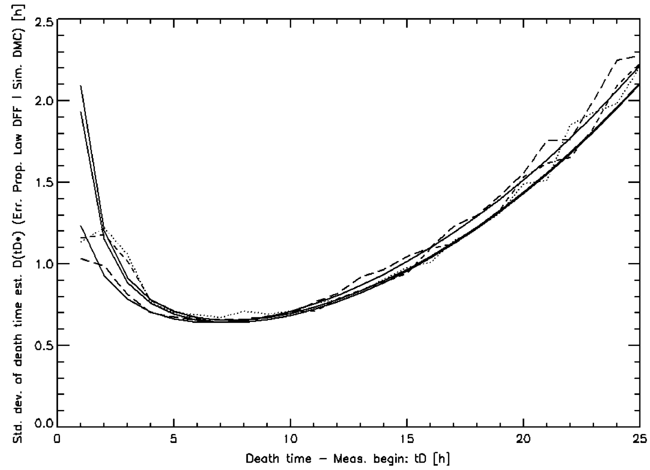

Fig. 2 Case (b): m=80 kg. Standard deviation D_{EP} (*solid lines*) and D_{MC} (*dashed lines*) as functions of time t_D between death and measurement under the overall influence of the confounders T_0, T_E, T_k, and t_k. For MC simulations, the numbers, N, of measurements are: N=1 (*dotted*), N=10 (*small dashes*), N=100 (*long dashes*)

t_D=1.3 h in case (a), t_D=1.0 h and t_D=2.1 h in case (b), and t_D=1.3 h and t_D=3.2 h in case (c), followed by a smooth and concave course during t_D=2 h and t_D=12 h (phase II) with standard deviations D between D=0.5 h and D=0.8 h in case (a), D=0.7 h and D=1.8 h in case (b), and D=0.8 h and D=2.0 h in case (c). In the third phase (III) between t_D=12 h and t_D=25 h postmortem, the standard deviations D show an exponential increase with values from D=0.7 h up to D=3.4 h in case (a), from D=0.8 h up to D=2.3 h in case (b), and from D=0.9 h up to D=1.9 h in case (c). By comparing the simulated temperature curve in Fig. 1 with the three phases of the standard deviations described, phase II (t_D=2–12 h) corresponds to the steepest temperature

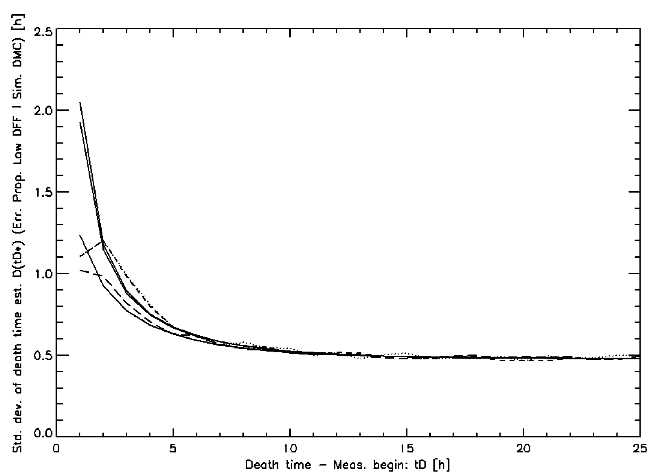

Fig. 3 Case (b): m=80 kg. Standard deviation D_{EP} (*solid lines*) and D_{MC} (*dashed lines*) as functions of time t_D between death and measurement under the isolated influence of the confounder T_0. For MC simulations, the numbers, N, of measurements are: N=1 (*dotted*), N=10 (*small dashes*), N=100 (*long dashes*)

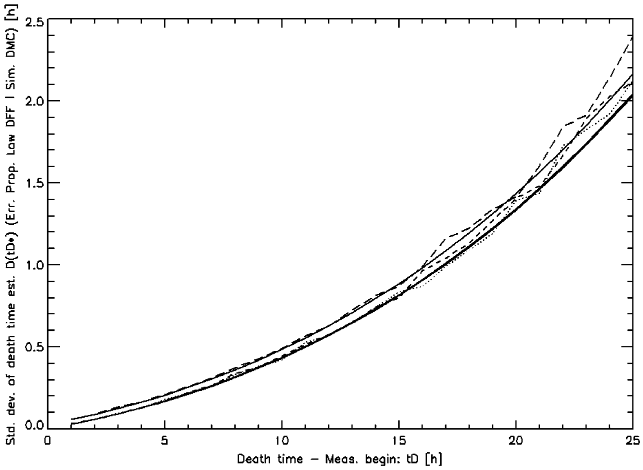

Fig. 4 Case (b): m=80 kg. Standard deviation D_{EP} (*solid lines*) and D_{MC} (*dashed lines*) as functions of time t_D between death and measurement under the isolated influence of the confounder T_E. For MC simulations, the numbers, N, of measurements are: N=1 (*dotted*), N=10 (*small dashes*), N=100 (*long dashes*)

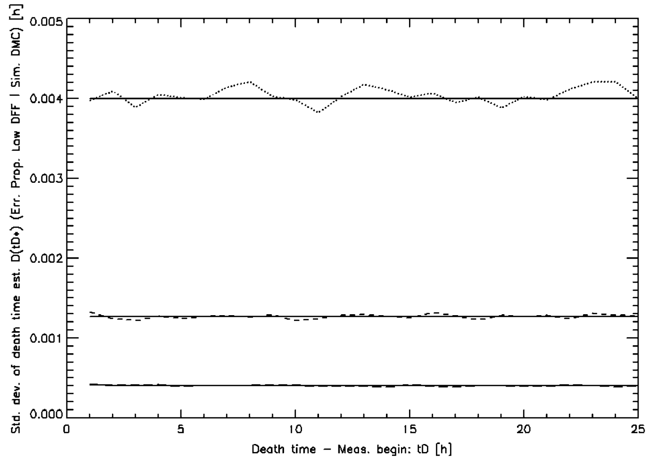

Fig. 6 Case (b): m=80 kg. Standard deviation D_{EP} (*solid lines*) and D_{MC} (*dashed lines*) as functions of time t_D between death and measurement under the isolated influence of the confounder t_k. For MC simulations, the numbers, N, of measurements are: N=1 (*dotted*), N=10 (*small dashes*), N=100 (*long dashes*)

decrease. Phase I represents the temperature plateau observed during the first 2 h postmortem and phase III the asymptotic part of the temperature curve converging to the ambient temperature T_E. The order of the curves at time t_D=1 h is $D_{EP,1} < D_{EP,10} < D_{EP,100}$. With increasing ordinate t, this order reverses so that at time t_D=25 h since death, $D_{EP,1} > D_{EP,10} > D_{EP,100}$.

Discrepancies between the Monte Carlo standard deviation D_{MC} and the standard deviation D_{EP} obtained by the law of error propagation for very short and for high values of t_D are caused by the intrinsic error of cutting off the Taylor series expansion at order 1 in the error propagation computations.

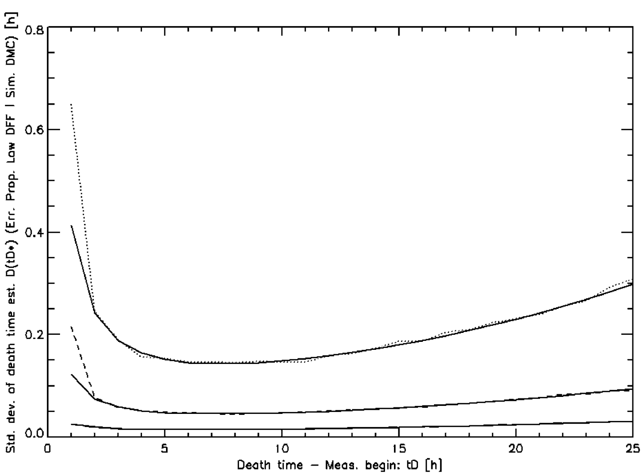

Fig. 5 Case (b): m=80 kg. Standard deviation D_{EP} (*solid lines*) and D_{MC} (*dashed lines*) as functions of time t_D between death and measurement under the isolated influence of the confounder T_k. For MC simulations, the numbers, N, of measurements are: N=1 (*dotted*), N=10 (*small dashes*), N=100 (*long dashes*)

Isolated influence of errors in the input parameters

Figure 3 presents the isolated influence of deviations of the initial temperature at the time of death T_0 (±0.5°C) for case (b). The standard deviations of all cases (a), (b), and (c) are given in Table 2. All standard deviations D_{EP} and D_{MC} show similar functional graphs. The maximum standard deviation of the death time estimate is observed at the beginning of the cooling process: D=0.7–1.3 h in case (a), D=0.5–2.0 h in case (b), and D=0.6–3.0 h in case (c). It then decreases and reaches a constant level after 5 h with a standard deviation of about D=0.4 h in case (a), D=0.5 h in case (b), and D=0.6 h in case (c). The accordance between D_{MC} and D_{EP} is good except for the beginning of the cooling process. The curves $D_{EP,1}(t_D^*)$ and $D_{EP,10}(t_D^*)$ are almost identical, while the curve $D_{MC,100}(t_D^*)$ assumes smaller values in the beginning of the cooling process.

Figure 4 presents the isolated influence of deviations or measurement errors of the environmental temperature T_E (±1°C), the graph shows $D(t_D)$ for case (b), while the data of all three cases are presented in Table 3. All curves indicating the development of the standard deviation of the death time estimate whether calculated by the error propagation law (D_{EP}) or computed by means of Monte Carlo simulation (D_{MC}) are in good correspondence and rise exponentially with rising t_D values (D=0 for t_D=0 h and D=2.7–3.3 h for case (a), D=2.0–2.3 h for case (b), and D=1.6–1.8 h for case (c) at t_D=25 h). For t_D values in the interval (15–25 h), the D_{MC} curves show slightly higher values than the D_{EP} curves.

Figure 5 presents the influence of measurement errors of the rectal temperatures T_k (±0.1°C) in case (b). The data of all three cases are given in Table 4. All the functions show a concave shape monotonically falling in the t_D interval (0–5 h) in case (a), in the interval (0–6 h) in case (b), and in the interval (0–7 h) in case (c) and monotonically rising afterwards. For $N=1$ and $N=10$, the function values $D_{EP,1}(t_D^*)$ and $D_{EP,10}(t_D^*)$ significantly differ from the respective values $D_{MC,1}(t_D^*)$ and $D_{MC,10}(t_D^*)$ only in the beginning of the cooling process up to $t_D=2$ h. For $N=100$, the curves D_{EP} and D_{MC} match perfectly. The graphs show that in a time range of $t_D=1$–25 h, a maximal improvement of 0.3 h in standard deviation can be achieved by augmenting the number of rectal temperature measurements (t_k,T_k).

Figure 6 presents the influence of errors in time measurement t_k (±14.4 s) for case (b). The standard deviations $D(t_D)$ of all cases are presented in Table 5. Since the numerical differences among cases (a), (b), and (c) lie in an order of magnitude of 10^{-5}h, Table 5 was not diversified to contain the special cases (a), (b), and (c), but the numbers were rounded accordingly. As expected, the isolated influence of time measurement errors on death time determination can be described as simply passing the unchanged error to the death time estimator. The functional graphs of D_{EP} and D_{MC} show constant standard deviations (for $N=1$: $D=0.004$ h, for $N=10$: $D=0.00126$ h, for $N=100$: $D=0.0004$ h). In accordance with our expectations, the accuracy $D(t_D^*)$ of time determination is inversely proportional to the number N of measurements since: $D_1 := D(t_D^*)_{N=1} = 0.004$ h, $D_{10} := D(t_D^*)_{N=10} = D_1/(10)^{1/2} = 0.00126$ h, $D_{100} := D(t_D^*)_{N=100} = D_1/(100)^{1/2} = 0.0004$ h. As further expected, $D(t_D^*)$ does not depend on the true time of death t_D or the body mass m at all.

Discussion

Since temperature-based methods represent essential tools in forensic death time determination, error control is an issue of major interest. The fact that only few studies have dealt with that problem so far (e.g., [2]) has to be attributed to numerous difficulties inherent in the subject. Experimental approaches need much organizational effort, are expensive, and compromised by ethical problems. More theoretically oriented projects run into some non-trivial problems concerning physics, statistics, and mathematics.

The present study follows a theoretical approach by investigating error control of the well-known model of Marshall and Hoare (see Eqs. 2.5.1–2.5.5) with the widely used parameter function (see Eqs. 2.5.9–2.5.11) established by Henssge. The approach is performed in two different and independent ways: by a Monte Carlo simulation and by a purely mathematical derivation of formulae for the standard deviation of the variable under investigation. This binary approach helps to gain control over the typical weaknesses of each of the two procedures: The Monte Carlo method is frequently subject to skeptical inquiries about the size of the simulated samples as well as about the goodness of the random generators used. The mathematical derivation of the formulae has to use certain assumptions for approximations. By comparing the results of those two approaches, inherent deficiencies of the approaches are compensated.

In contrast to Henssge's error analysis (e.g., [3]) or to the well-known death time computing nomogram (e.g., [17]), our results do not prove a simple functional dependence from the so-called normalized temperature $Q := (T - T_E)/(T_0 - T_E)$ − which, in case of [17], is: $1.0 > Q \geq 0.5$: $D =$ approx. 1.3 h; $0.5 > Q \geq 0.3$: $D =$ approx. 1.5 h; $0.3 > Q \geq 0.2$: $D =$ approx. 2 h—but a clear dependence from the absolute time since death t_D instead. This fact can be explained by looking at the graph of $D(t_D^*)$ as a function of t_D, e.g., in Fig. 2. The error $D(t_D)$ is rather small in regions where the rectal temperature–time curve $T(t)$ has a steep descent—the region between approx. 3 and 13 h postmortem—whereas in regions with slow descent—in the initial plateau phase and in the final asymptotic phase with $T(t)$ exponentially approaching T_E—the errors are much bigger in comparison. This finding is plausible when it is interpreted in the context of the shift algorithm (Eq. 1.2) which requires matching the measured point(s) in the time–temperature coordinate system to the model curve by shifting the latter along the t-axis and seeking the location of the minimal distance between measured points and model curve. This interpretation becomes clear by referring to a hypothetical scenario where the model curve is a straight line with a fixed descent: The possible error is much greater if the line runs almost parallel to the t-axis than if the line has a steep descent.

It is of some interest to compare our results to two studies dealing with the subject of controlling measurement errors in temperature-based death time determination. In a great multicenter study [16] of 1990 which presents 76 = 46 (case group I) + 30 (case group II) cases from six university institutes of forensic medicine in Germany, the authors presented the diagram (Abb. 3 in [16]) of a histogram which is an empirical approximation to the probability density of an analogon of our Δt_D. Since this diagram does not take into account the dependence of the probability density function of Δt_D on time since death t_D, it has to be regarded as an average diagram. The standard deviation $D(\Delta t_D^*)$ which is identical to $D(t_D^*)$ is computed by the authors as 1.26 h. Since most of the cases of group I in the study represented death times of $t_D \leq 16$ h, the findings are in reasonable correspondence with our Fig. 2 and Table 3.

In the study [2], the authors used a Newtonian model (which is a simple exponential approach to body cooling) and applied it to the temperature of the external auditory canal of a body cooling under controlled conditions in a refrigerated room of 4°C. They investigated the errors in death time back-calculation based on their model given (a) two temperature–time points with a time difference of 30 min or, alternatively, (b) 30 temperature–time points with a time difference of 1 min between each pair of two successive points and found that the rounding errors caused by a thermometer resolution of 0.1°C may significantly influence death time estimation. During the 16-h course of cooling, they noticed relative back-calculation errors $e(t_D)$: = $|t_D{}^* - t_D|/t_D$ which ranged from (a): $e(t_D)=0.6$ and (b): $e(t_D)=0.4$ in the first hours postmortem up to (a): $e(t_D)=2.4$ and (b): $e(t_D)=1.8$ after 16 h of cooling. Additionally, the authors simulated cooling curves with specific artificially added rounding errors and investigated the isolated effect of rounding by computing analogous back-calculations to (a) and (b) based on their simulated data. Though the rounding turned out to be not fully responsible for the overall error, it considerably added (approx. two thirds of the total error) to the empirical error observed in case (a), whereas in case (b), the relative error $e(t_D)$ was reduced to a range of (0.9–1.1). Due to the special cooling case and experimental method as well as to the different mathematical model, the results of this study cannot be directly compared to our results. However, two significant qualitative facts in both results may be emphasized:

1. Temperature measurement errors which seem insignificant may cause significant errors in death time back-calculation.
2. Improving measurement accuracy is a major goal in temperature-based death time calculation.

Few authors followed the idea of yielding better results by using exponential models of higher order (e.g., [18]). Since those models are known in statistics to notoriously cause numerical problems, we do not discuss the findings based on triple exponential models here.

One effect which can be seen in Table 3 is the influence of the body mass on the part of the standard deviation $D(t_D{}^*)$ of the death time estimator $t_D{}^*$ related to the error $D(T_E)$ of the environmental temperature T_E (see Table 1 for the overall standard deviation, respectively). The standard deviation D_{EP} at $t_D=25$ h shows falling values with rising body mass: $m=60$ kg (case (a)): $D_{EP}=1.67$ h (isolated T_E (Table 3, $N=1$)), $D_{EP}=1.80$ h (all factors (Table 1, $N=1$)); $m=80$ kg (case (b)): $D_{EP}=2.04$ h (isolated T_E (Table 3, $N=1$)), $D_{EP}=2.12$ h (all factors (Table 1, $N=1$)); $m=100$ kg (case (c)): $D_{EP}=2.72$ h (isolated T_E (Table 3, $N=1$)), $D_{EP}=2.77$ h (all factors (Table 1, $N=1$)). An analogous effect can be observed in cases of N temperature measurements values with $N=10$ and $N=100$ in D_{EP} as well as in D_{MC}. This finding is explained by the fact that the standard deviation D in the third phase of cooling, which is the asymptotic descent of T to the environmental temperature T_E, is almost completely governed by the exponential rising of the part $D(T_E)$ of the overall standard deviation D with time t_D. Since a body of bigger mass m enters this third phase of cooling later than a body of lower mass m' (see Fig. 1), the comparison of D for different body masses $m > m'$ at a fixed point t_D in time, which is chosen sufficiently late in the cooling process (as $t_D=25$ h in our example above), leads to the result given above. The "shifting" of the cooling curve $T(t)$ with rising body mass m can be illustrated evidently by looking at the so-called normalized temperature Θ: = $(T - T_E)/(T_0 - T_E)$ taken from Fig. 1: At $t_D=25$ h, one obtains the following different values for rising body mass: $m=60$ kg (case (a)): $\Theta=4/19.2 = 0.21$; $m=80$ kg (case (b)): $\Theta=6/19.2 = 0.31$; $m = 100$ kg (case (c)): $\Theta=8/19.2 = 0.41$. This means that while case (a) is in the midst of the third (asymptotic) cooling phase at $t_D=25$ h (indicated by the small value of Θ), case (c) is just passing the second cooling phase at $t_D=25$ h with the steepest temperature descent. Actually, the value $\Theta=0.2$ is recommended as lower limit for the applicability of the Henssge nomogram method. This particular aspect of our results is discussed in detail to prevent the reader from concluding that a higher body mass m generally leads to a smaller standard deviation D in death time determination.

Our results on input errors in the Henssge model can be summarized and commented as follows:

- The number N of measurements (t_k,T_k) $k=1,\ldots,N$ is of minor influence on the standard deviation D of the death time estimate only. In almost all functional dependencies investigated, the variation of D was significantly smaller than 1 h, whereas the parameter N varied from $N=1$ up to $N=100$. This indicates that death time determination cannot be improved by simply increasing the number of successive measurements (t_k,T_k).
- The influence of deviations of the initial rectal temperature T_0 at the time of death (see Fig. 3) remains constant at $D=0.5$–0.7 h after approx. 5 h postmortem. This result supports intuition that "in the long run," an error in the initial rectal temperature should have a simple additive effect on the result of the death time estimation, while shortly after death, errors in the initial (start) temperature T_0 clearly dominate all other errors. The numerical results indicate that the influence of errors in the initial temperature of ±0.5°C, which cannot be avoided, can be tolerated.
- Relatively small errors of the environmental temperature T_E of ±1°C (see Fig. 4) exert an immense influence on the standard deviation of the death time estimate. Starting from $D=0$ h at $t_D=0$ h, the standard deviation increases exponentially with increasing time difference

t_D between death and measurement begin. Errors in the environmental temperature dominate all other input data errors at times t_D>5 h postmortem. This result corresponds to intuition that errors of the assumed environmental temperature accumulate with time t_D and can increase without upper limit. The result also demonstrates that careful measurement or estimation of the environmental temperature T_E is one of the most important issues of death time determination.

– The functional dependence of death time standard deviation D of measurement errors of the rectal temperatures T_k (see Fig. 5) taken at times t_k is similar to that of the combined influence of all input variables (see Fig. 2), but the values of the standard deviations of the death time estimate are much smaller (only 1/10 (for N=1) to 1/100 (for N=100)). It is generally accepted that the error of death time determination due to postmortem rectal temperature measurement can be significantly reduced by taking more measurements. The presented calculations show that the error attributable to rectal temperature recording accounts only for a comparatively small part of one tenth to one hundredth of the overall error in death time estimation. Thus, the effect of multiple rectal temperature measurements might remain negligibly small.
– As expected, the errors (0.004 h) in the time measurements t_k (see Fig. 6) taken simultaneously with temperature measurements T_k are passed unchanged onto the standard deviation D of death time determination (0.004 h for N=10 to 0.0003 h for N=100; note that 0.0004 h≈0.004 h/$(100)^{1/2}$), and this dependency is constant throughout the cooling process. Taking more measurements can still reduce this comparably small part of the overall error.

In summary, two theoretically oriented approaches of controlling the influence of errors in the input data on forensic death time determination are presented. The results of both methods correspond very well and thus indicate that both methods are valid.

Of the variables investigated, two, the initial rectal temperature at death T_0 and environmental temperature T_E, were identified where errors considerably contribute to the overall error in estimating death time. Errors of the initial rectal temperature T_0 dominate the beginning of the cooling process, while errors of the environmental temperature T_E exponentially increase the standard deviation of the death time estimate throughout the later cooling process. The influences of the errors of other variables (rectal temperature, time) proved to be nearly negligible. Series of rectal temperature recordings do not significantly improve the death time estimation compared to the common single temperature recording at the crime scene.

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